

Nucleon and Δ in a covariant quark-diquark model

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Abstract

We develop a formally covariant quark-diquark model of the nucleon. The nucleon is treated as a bound state of a constituent quark and a diquark interacting via a quark exchange. We include both scalar and axial-vector diquarks. The underlying Bethe-Salpeter equation is transformed into a pair of coupled Salpeter equations. The space- and timelike electromagnetic form factors of the nucleon are calculated in the Mandelstam formalism for squared momentum transfers up to 3 (GeV/c)^2 above threshold. The model is applied also to the $\Delta(3/2)$.

1 Introduction

Many attempts have been made to describe the nucleon as a bound state of a quark and a diquark. The most fundamental approach uses the Nambu–Jona-Lasinio model^{1,2,3} to derive static properties of the nucleon and ground state baryons. Most diquark models favour the existence of two kinds of diquarks, scalar and axial-vector ones. To take into account the truly three quark structure and the Pauli-principle a quark exchange between quark and diquark is assumed to dominate the short range interaction³. We adopt these ideas in a constituent quark model based on the Bethe-Salpeter equation. Following Salpeter⁴ we assume an instantaneous interaction and obtain a Salpeter-type equation. As the only interaction we consider an (instantaneous) quark exchange between the quark and the diquark. Involving only scalar and axial-vector diquark channels we deduce a pair of coupled integral equations similar to the framework of Ref.⁵. Within a basis of positive parity amplitudes with spin-1/2 we obtain a bound state solution for this equation by making use of the Ritz variational principle. Then, electromagnetic transition currents are calculated using the Mandelstam formalism⁶. For details of the calculation see two recent papers^{7,8}. We compute the electromagnetic form factors of the nucleon in both the space- and timelike regions. Extending the model to the $\Delta(3/2)$ resonance, the $N - \Delta$ transition form factors and the ratios $E2/M1$ and $C2/M1$ are calculated.

2 The model

In momentum space the *Bethe-Salpeter amplitude* for a bound state of a quark (index 2) and a scalar or axial-vector diquark (index 1) with total four momentum $P = p_1 + p_2$ and relative momentum p fulfills the *Bethe-Salpeter equation*:

$$\chi_P(p)_{(\mu)} = \Delta_1(p_1)_{(\mu)}^{(\nu)} S_2(p_2) \int \frac{d^4 p'}{(2\pi)^4} (-iK(P, p, p') \chi_P(p'))_{(\nu)} . \quad (1)$$

Δ_1 and S_2 are the propagators of the constituent diquark and the quark, respectively. For the v-diquark we choose $\Delta_{\mu\nu} = -ig_{\mu\nu}/(m_1^2 - p_1^2 + i\varepsilon)$. Following Salpeter⁴ we neglect the time (i.e. energy) dependence of the interaction kernel by assuming $K(P, p, p') = V(\vec{p}, \vec{p}')$ (*instantaneous interaction*) in the rest frame of the bound state. Then, one can easily perform the p^0 integration. Defining in the rest frame of the bound state the *Salpeter amplitude*

$$\Phi(\vec{p}) := \left(\int \frac{dp^0}{2\pi} \chi_P(p^0, \vec{p}) \right)_{P=(M, \vec{0})} , \quad (2)$$

one gets from Eq. (1) with standard techniques:

$$\Phi(\vec{p}) = \frac{1}{2\omega_1} \left(\frac{\Lambda_2^+(-\vec{p})\gamma^0}{M - \omega_1 - \omega_2} + \frac{\Lambda_2^-(-\vec{p})\gamma^0}{M + \omega_1 + \omega_2} \right) \int \frac{d^3 p'}{(2\pi)^3} V(\vec{p}, \vec{p}') \Phi(\vec{p}') , \quad (3)$$

with $\Lambda_2^\pm(-\vec{p}) = \frac{\omega_2 \pm H_2(-\vec{p})}{2\omega_2}$ the usual Dirac projectors and $\omega_i = \sqrt{\vec{p}_i^2 + m_i^2}$. If we define

$$\Psi(\vec{p}) := \gamma^0 \Phi(\vec{p}) , \quad W(\vec{p}, \vec{p}') := V(\vec{p}, \vec{p}') \gamma^0 , \quad (4)$$

we can rewrite Eq. (3) in a Schrödinger-type equation:

$$\begin{aligned} (\mathcal{H}\Psi)(\vec{p}) &= M \Psi(\vec{p}) \\ &= \frac{\omega_1 + \omega_2}{\omega_2} H_2(\vec{p}) \Psi(\vec{p}) + \frac{1}{2\omega_1} \int \frac{d^3 p'}{(2\pi)^3} W(\vec{p}, \vec{p}') \Psi(\vec{p}') \\ &=: (\mathcal{T} + \mathcal{V}) \Psi(\vec{p}) . \end{aligned} \quad (5)$$

Of special importance in the Bethe-Salpeter approach is the correct normalization of the amplitudes. This will guarantee the correct behaviour of the form factors at $q^2 = 0$. In the instantaneous approximation the usual *normalization condition* for the BS-amplitudes leads to

$$\int \frac{d^3 p}{(2\pi)^3} (2\omega_1) \bar{\Phi}(\vec{p}) \gamma^0 \Phi(\vec{p}) = 2M , \quad (6)$$

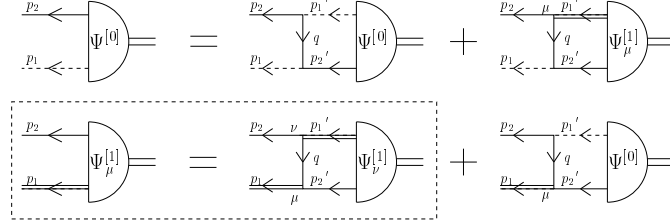


Figure 1: The quark-diquark Salpeter equations. The framed part is the equation for the $\Delta(3/2)$ (see Sec. 4).

with $\bar{\Phi}$ the adjoint Salpeter amplitude, fulfilling $\bar{\Phi}(\vec{p}) = \Phi^\dagger(\vec{p})\gamma^0$. This leads to the definition of a *scalar product* for $\Phi(\vec{p})$:

$$\langle \Phi_1 | \Phi_2 \rangle = \int \frac{d^3p}{(2\pi)^3} (2\omega_1) \bar{\Phi}_1(\vec{p}) \gamma^0 \Phi_2(\vec{p}) . \quad (7)$$

For $M \in \mathcal{R}^+$, the Hamiltonian \mathcal{H} (Eq. (5)) has to be hermitian (and positive definite) with respect to the above scalar product. In the instantaneous approximation the interaction kernel is of the form

$$W(\vec{p}, \vec{p}') \sim -g^2 \frac{1}{\omega_q^2} (-\vec{\gamma}(\vec{p} + \vec{p}') + m_q) \gamma^0 , \quad (8)$$

with ω_q the energy of the exchanged quark and g the dimensionless quark-diquark coupling parameter. To obtain a stable solution of the Salpeter equation (5) we have to introduce a form factor of the diquark. It is chosen to be of the form $\sim \exp(-\lambda^2 k^2)$, with λ parametrizing the extension of the diquark. Transitions between scalar and v-diquarks lead to a system of coupled Salpeter equations shown graphically in Fig. 1.

3 Nucleon form factors

The electromagnetic current is the sum of the diquark- and quark currents, see Fig. 2. In the *Mandelstam formalism* one finds e.g. for the quark current:

$$\langle P' s' | j_\mu^q | P s \rangle = e_q \int \frac{d^4p}{(2\pi)^4} \bar{\Gamma}_{P'}(p') S_2^F(p'_2) \gamma_\mu S_2^F(p_2) \Gamma_P(p) \Delta_1^F(p_1), \quad (9)$$

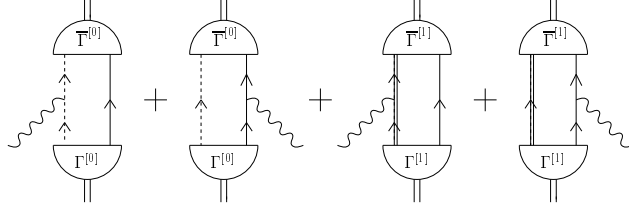


Figure 2: The electromagnetic current is the sum of the diquark currents and the quark currents.

	m_q	$m_S = m_V$	g^N	g^Δ	λ	κ_V	κ_{SV}
Set A	440 MeV	800 MeV	17.76	8.50	0.30 fm	1.1	-
Set B	440 MeV	800 MeV	17.76	8.50	0.30 fm	-0.07	2.4

Table 1: The parameters of the model.

with the vertex function $\Gamma_{P=(M,\vec{0})}(p) = \Gamma(\vec{p}) = -i \int \frac{d^3 p'}{(2\pi)^3} V(\vec{p}, \vec{p}') \Phi(\vec{p}')$ in the instantaneous approximation. The form factors are obtained from:

$$J_\mu := e \bar{u}_{s'}(P') \left(\gamma_\mu F_1^N(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2M} \kappa_N F_2^N(q^2) \right) u_s(P), \quad (10)$$

with $q = P' - P$. The parameters used are given in Tab. 1. The scalar and v-diquark parameters are chosen to be equal. g^N is the quark-diquark coupling parameter for the nucleon (cf. Eq. (8)) and g^Δ that for the Δ (see Sec. 4). κ_V is the anomalous magnetic moment of the v-diquark introduced via

$$j_{\mu;ba}^{V-V} \sim -(p+p')_\mu g_{ba} + (1+\kappa_V)(p_b - p'_b)g_{\mu a} + (1+\kappa_V)(p'_a - p_a)g_{\mu b}. \quad (11)$$

We compare the results of the parameter Set A with those of Set B where we allow also for photon-induced scalar-v-diquark transitions according to

$$j_\mu^{S-V} \sim \frac{(1+\kappa_{SV})}{M_N} \varepsilon_{\mu\nu\rho\lambda} \epsilon^\nu p_2'^\rho p_2^\lambda. \quad (12)$$

Fig. 3 shows the Sachs form factors of the nucleon in the spacelike region. Note that κ_V and κ_{SV} only affect the spin-flip currents, i.e. the magnetic form factors. We find a very good description of all form factors up to $-q^2 = 3 \text{ GeV}^2$. The resulting static properties are listed in Tab. 2. The inclusion of scalar-v-diquark transitions is a possibility to describe also the neutron magnetic

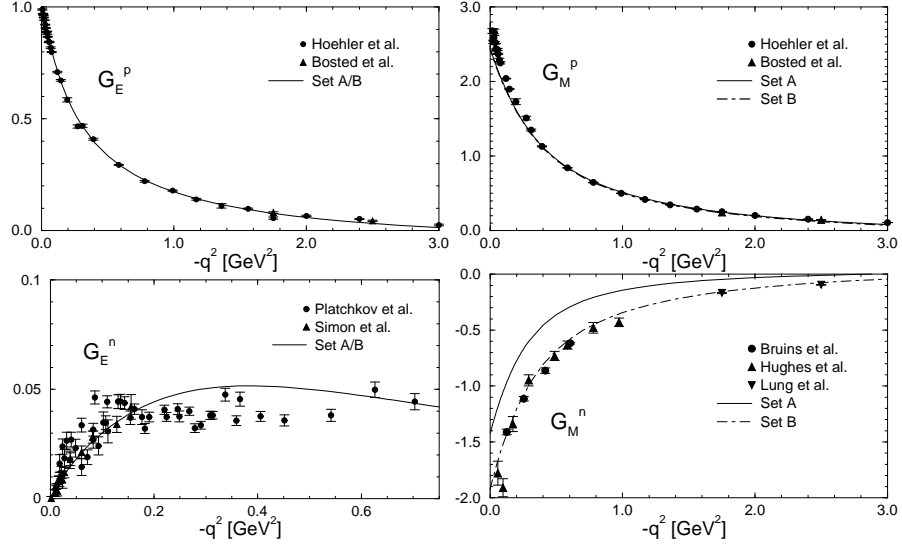


Figure 3: The spacelike Sachs form factors of the nucleon. For the experimental data see the analysis of Ref. ⁹.

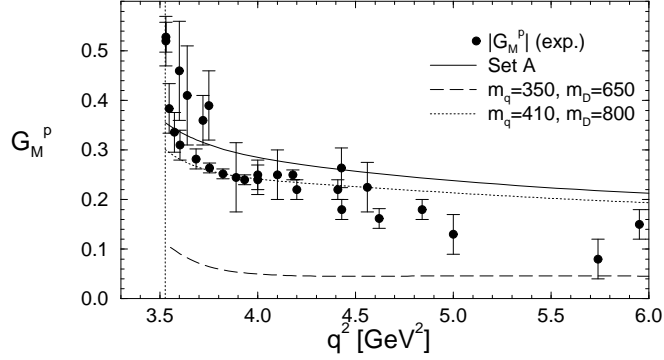


Figure 4: The timelike magnetic form factor of the proton. For the experimental data see Refs. ^{10,11}.

	$\sqrt{\langle r^2 \rangle_E^p}$	$\langle r^2 \rangle_E^n$	$\sqrt{\langle r^2 \rangle_M^p}$	$\sqrt{\langle r^2 \rangle_M^n}$	μ_p/μ_N	μ_n/μ_N
Set A	0.79 fm	-0.110 fm ²	0.72 fm	0.86 fm	2.45	-1.42
Set B	0.79 fm	-0.110 fm ²	0.74 fm	0.75 fm	2.44	-1.91
exp.	0.847 fm	-0.113 fm ²	0.836 fm	0.889 fm	2.793	-1.913

Table 2: Static nucleon properties as they result from the threshold behaviour of the electromagnetic nucleon form factors. For the experimental data see the analysis of Ref. ⁹.

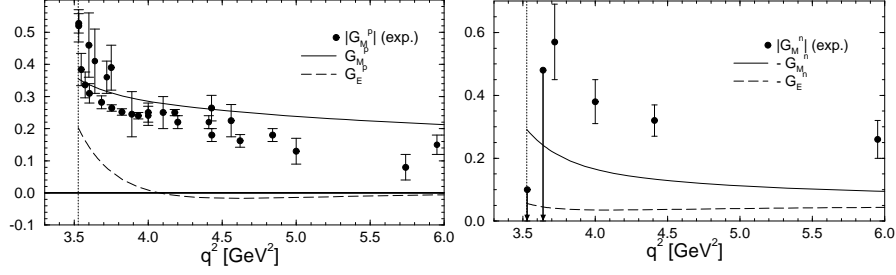


Figure 5: The timelike magnetic and electric form factors of the proton (left panel) and of the neutron (right panel). For the experimental data see Refs. ^{10,11}.

form factor. Fig. 4 shows the proton magnetic form factor in the timelike region. We find that the threshold value is very mass dependent, see the dashed and dotted curves. The dotted curve corresponds to a best fit to the electromagnetic form factors in the space- and timelike region. The masses, however, lie below the $\Delta(1232)$ threshold. The definition of the Sachs form factors leads to the threshold conditions:

$$G_E(4M^2) = G_M(4M^2) \leftrightarrow J_+(4M^2) = 2M \frac{d}{dP'} J_0(4M^2), \quad (13)$$

with $P' = |\vec{P}'| = q$, $P = (M, \vec{0})$ and $J_+ = 1/2(J_1 + iJ_2)$. Fig. 5 shows $G_{E,M}^{n,p}$. We find that Eq. (13) is violated in our model. This is due to the choice of the propagator of the v -diquark. Considering the second diagram in Fig. 2 alone yields the result of Fig. 6, which shows that the relativistic treatment is correct.

4 $N - \Delta$ transition form factors

In our model, the $\Delta(3/2)$ appears as a bound state of a v -diquark and a quark, see the framed part in Fig. 1. To fix the Δ mass at 1232 MeV we introduce

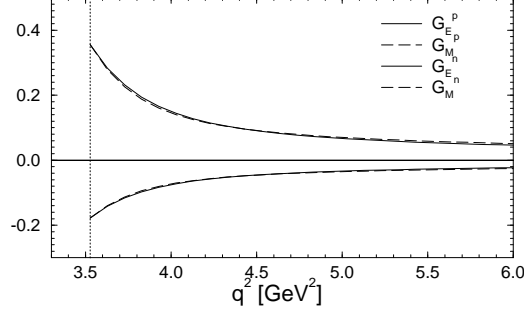


Figure 6: The timelike nucleon form factors as they result from the quark current with a scalar diquark as spectator *alone*. The upper two curves are the proton form factors, the negative ones are those of the neutron.

the quark-diquark coupling parameter g^Δ , see Tab. 1. The Δ amplitude is normalized according to $\langle \Psi_\Delta | \Psi_\Delta \rangle = 2M_\Delta$. The electromagnetic $N - \Delta$ transition is decomposed via¹²

$$e J_\mu(q^2) = e \sqrt{2/3} \bar{u}_{s'}^\beta(P') J_{\beta\mu} u_s(P), \quad (14)$$

$$\text{with } J_{\beta\mu} = G_M(q^2) G_{\beta\mu}^M + G_E(q^2) G_{\beta\mu}^E + G_C(q^2) G_{\beta\mu}^C, \quad (15)$$

with G_M, G_E, G_C the magnetic dipole, electric and Coulomb quadrupole form factors, respectively. In the rest frame of the incoming nucleon this leads to:

$$\begin{pmatrix} G_M(q^2) \\ G_E(q^2) \end{pmatrix} = \sqrt{3/2} \frac{2\sqrt{2}}{g(q^2)} \begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{12} \\ \frac{1}{4\sqrt{3}} & -\frac{1}{12} \end{pmatrix} \begin{pmatrix} J_+(q^2) \\ J'_+(q^2) \end{pmatrix} \quad (16)$$

$$G_C(q^2) = -\sqrt{3/2} \frac{4M_\Delta M_N}{g(q^2) \sqrt{6Q^+ Q^-}} J_0(q^2), \quad (17)$$

with $Q^\pm = (M_\Delta \pm M_N)^2 - q^2$, $g(q^2) = ((M_\Delta + M_N)/(2M_N))\sqrt{Q^-}$ and $J'_+ = \langle +3/2 | J_+ | +1/2 \rangle$. Fig. 7 shows the calculated form factors and the experimental G_M ¹³. We find that G_M comes out a factor of 2 too low. Here, the inclusion of scalar to v-diquark transitions leads only to a small improvement. Note that G_C is negative, and $G_E(q_s^2) \approx G_C(q_s^2) \approx 0$ at the pseudothreshold $q_s^2 = (M_\Delta - M_N)^2$. Figs. 8 and 9 show the ratio $E2/M1$ and the absolute value $|C2/M1|$, respectively. With Set B we find the correct threshold value $E2/M1 = -3.4\%$ ¹⁴. For $C2/M1$ we obtain a positive sign, with the absolute value describing the data well. Note that we follow consistently the definitions

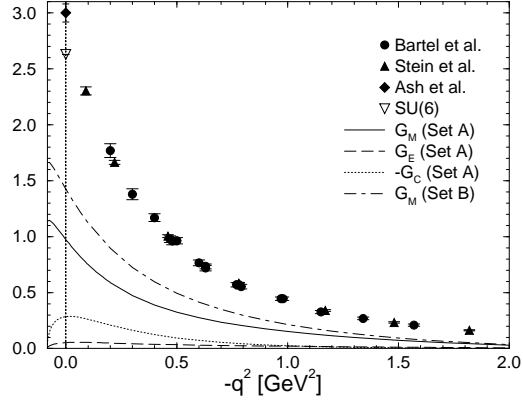


Figure 7: The N-Δ transition form factors.

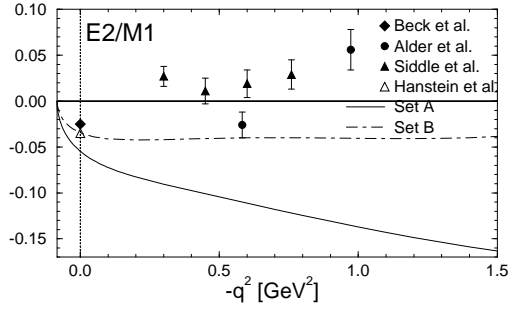


Figure 8: The ratio $E2/M1$.

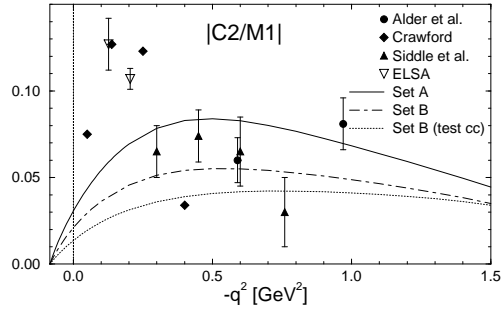


Figure 9: The ratio $|C2/M1|$.

of Ref. ¹². The two lines of Set B indicate the variance of the calculated ratio due to the fact that the $N - \Delta$ current is only partially conserved.

5 Summary

We developed a formally covariant quark-diquark model of the nucleon. The nucleon is assumed to be composed of a quark and a scalar/axial-vector diquark which interact via an instantaneous quark exchange. With a single set of parameters we are able to describe the nucleon electromagnetic form factors in the space- and timelike region for momentum transfers up to 3 GeV^2 above threshold. In this picture, the $\Delta(3/2)$ is a bound state of a quark and a v-diquark. The results for the $N - \Delta$ transition agree only qualitatively with experiment, but we find the correct value for $E2/M1$.

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